Basic Astronomical Data

The length of the tropical year = 365.242199 days in 1900. At the present time it is slowly getting shorter.

The length of the synodic month = 29.5305882 days. At the present time it is also getting shorter very slowly, but tidal effects will cause it to grow steadily longer in the long run.

Julian Day Numbers

Astronomers keep a running count of days which began several thousand years in the past. This running day count is independent of all calendar systems, and can therefore be used as a convenient way to transform dates from one system to another. It also is a convenient way to define the starting points (first day of the first month of the year 1) of calendar systems. By convention, these days (called Julian Days which is unfortunate because they must not be confused with the Julian Calendar system) begin at noon. Thus calendar days, which in the Gregorian and related systems begin at midnight, usually have their Julian Day numbers equal to some integer plus 0.5. Given the Julian Day of any calendar date, the day of the week can easily be computed simply by dividing by seven and examining the remainder.

The zero point of the Julian Day system occurred at noon GMT on January 1, 4713 B.C. in the Julian Calendar, or November 24, 4714 B.C. in the Gregorian calendar. When astronomers use the Julian day, the fractional part is meaningful and all Julian Day numbers are assumed to refer to GMT (more technically, to a quantity called Universal Time which is very close to GMT). For calendrical purposes, differences of time zones are generally ignored—that is, at each location a given date starts at midnight local time (Gregorian and its relatives, Elliott Super) or sunset local time (Jewish and Islamic).

General Calendrical Rules

1) The infinite range of years is divided into two *eras*, each of which is sometimes given a common abbreviation (e.g., B.C. and A.D.)

2) There is never a year 0. Positive-numbered years are in one era; negative-numbered years are in the other. The abbreviation for the era tells the sign of the year. Actual plus and minus signs are normally not used. (This rule is due to the fact that modern calendars are all evolved from earlier ones which were developed before the concepts of zero and negative integers were well understood.)

3) The sequence of weekdays (Sunday…Saturday) is assumed (by InterCal) to extend indefinitely into both the past and future without

interruption. Babylonians developed the seven-day week. The emperor Constantine officially adopted it for use in the Roman Empire in the fourth century, although it had been in common use for a very long time before that. The rules of most calendars are completely independent of this ever-repeating seven-day cycle. The Jewish calendar is an exception. Its rules prevent its New Year's Day, Rosh Hashonah, from occurring on certain days of the week.

4) Calendars are nothing more than day-counting systems. Months and years are merely convenient larger groupings of days. Usually, calendars do not depend on finer divisions of time (hours, minutes, etc.) in their rules. Once again, the Jewish calendar is an exception. The time of day of new moon nearest the start of a year can affect the date of Rosh Hashonah.

5) Calendars differ according to the significance given to the larger groupings (such as months or years). Although not all calendars fit into the following groups, all those implemented by InterCal **do**.

a) Lunar calendars—in lunar calendars the goal is to have the date within each month be a good indicator of the phase of the moon. Most lunar calendars have their months start and end as close to new moon as feasible. In such calendars, the first is near new moon, the eighth near first quarter, the sixteenth near full moon, and the twenty-third near third quarter. Thus it is desirable to have the average length of a month be 29.5305882 days (the synodic month—the average time between successive new moons). Years are just convenient larger groupings of months, and bear no particular relation to the motion of the earth-moon system around the sun.

b) Solar calendars—in solar calendars the goal is to have the date (month and day) be a good indicator of the season of the year. Thus it is desirable for the average length of a year to be 365.242199 days (the tropical year—the mean time between successive vernal equinoxes). Months bear no fixed relation to the phase of the moon.

c) Luni-solar calendars—in luni-solar calendars the object is to be both a lunar and a solar calendar at the same time. Naturally, this leads to more complex calendrical rules. The tropical year is nowhere close to an integer multiple of the synodic month. The first priority in luni-solar calendars usually is to keep accurate with respect to the moon. Extra months are added occasionally to achieve reasonable accuracy with respect to the sun (when averaged over several years).

6) Lunar, Solar, and Luni-Solar calendars differ according to whether their rules are **defined** in terms of the **actual** sun and/or moon, or **defined** in terms of **mathematical formulas** which are meant to approximate the sun and moon. I call the former type *physical* and the latter type *mathematical*. A **physical** calendar cannot actually be implemented with a finite computer algorithm. One must make approximations which mean that the error, as you go farther into the past

or future, cannot be quantified. The motions of the sun and moon are not known to infinite precision, and their orbits are not perfectly stable or computable over very long time frames. A **mathematical** calendar can be implemented with perfect accuracy (as far as its rules are concerned) on a computer, but of course over very long time spans it will become a poorer and poorer approximation to the sun and moon which it is supposed to track.

Calendar systems evolve over time. Rules are changed or standardized, and occasionally a new system replaces an older system. InterCal does not attempt to follow the various rules changes in any of its implemented calendars. In fact, for most ancient calendars there were few fixed rules and those that may have existed are largely unknown to modern scholars. InterCal simply extends the rules of each calendar in its present form indefinitely into both the past and the future. **The only exception is the "Western Historical Calendar", which is just a graft of the Gregorian Calendar onto the Julian (see below).** Thus, InterCal does not attempt historical accuracy, only mathematical accuracy.

Julian Calendar

Using the classification scheme above, the Julian Calendar is a mathematical solar calendar. It was instituted by Julius Caesar on January 1 (Julian) in the year which is now known as 45 B.C. It was a modification and improvement over an older calendar in use in Rome at the time. In order to get the calendar back in synchronism with the sun, Caesar lengthened the year 46 B.C. by several months. In his new system, January, March, May, July, September, and November had 31 days. April, June, August, October, and December had 30 days. February normally had 29, but had 30 in leap years. Augustus Caesar took a day from February and added it to August about 30 B.C. Actually, he also renamed the month, which had not been called August before. Additional rule changes took place during the next few decades. Also, the rules were not always correctly followed. The calendar reached its final form, with the month lengths being what we are used to, in 8 A.D. The Roman year at one time began in March, not January. The adoption of January as the first month occurred around 153 B.C. However, England was reckoning the start of the year at March 25 at the time it switched from the Julian calendar to the Gregorian (see below). So it is not quite correct to say that the Julian calendar was finalized in 8 A.D. There were still a few rule variations which varied from time to time and place to place. The Julian calendar was the official calendar of the Roman Empire and, later, the entire Christian world until its replacement by the Gregorian calendar.

There have been different numbering schemes for the days of February in leap years. One common method, adopted by the InterCal program, was to insert an unnumbered day (simply called the bisextile day) between February 24 and February 25.

The rules of the Julian calendar as implemented by InterCal are:

1) The negative year era is labeled B.C. (Before Christ). The positive year era is labeled A.D. (Anno Domini, Latin for Year of the Lord).

2) Months are named (in order) January, February, March, April, May, June, July, August, September, October, November, December. Years start in January.

3) The Julian Day of midnight at the start of January 1, 1 A.D. Julian (a Saturday) is 1721423.5.

4) January, March, May, July, August, October, and December have 31 days.

5) April, June, September, and November have 30 days.

6) February has 28 days in normal years and 29 days in leap years.

7) The leap year cycle is four years long. Leap years occur every fourth year, as follows:

…9 B.C., 5 B.C., 1 B.C., 4 A.D., 8 A.D, 12 A.D.… (Note that I do not simply say “years evenly divisible by 4” because that simplification of the rule only works for positive years. Given an unambiguous definition of the cycles, a more precise way of stating the rule would be: “Years **whose position in their cycle** is divisible by 4 are leap years.”)

8) The extra day in leap years is added between February 24 and February 25, is not numbered, and is referred to as “bisextile day”. **This rule was not standardized—other possibilities were used in various places and times. But this rule was common and is the one implemented by InterCal.**

With a leap year cycle of four years containing 4(365) + 1 days, the average year is 365.25 days, for an error of 0.0078 days per year, or about one day every 128 years.

Gregorian Calendar

The Gregorian Calendar is a slight modification of the Julian. So it is also a mathematical solar calendar. The differences between the Gregorian and Julian calendars involve the leap year rule, the zero point, and the numbering system of days in February of leap years.

The rule differences between Gregorian and Julian:

1) The Julian Day of midnight at the start of January 1, 1 A.D. Gregorian (a Monday) is 1721425.5.

2) The extra day in February of leap years is simply added at the end, becoming February 29.

3) Leap year cycles are 400 years long. One cycle began in 1 A.D. and ended in 400 A.D. The cycle preceding that one began in 400 B.C. and

ended in 1 B.C. Within any particular 400-year cycle, years **whose position in the cycle** is divisible by 4 but not by 100 **AND** the final (four-hundredth) year are leap years. As examples, 401 B.C., 1 B.C., and 2000 A.D. are leap years, but 501 B.C. and 1900 A.D. are not.

The Gregorian calendar was instituted by Pope Gregory XIII in 1582 A.D. It had been noticed that the seasons were moving earlier and earlier compared to given calendar dates. Equivalently, any fixed calendar date was occurring later and later in the year as defined by the seasons. It was clear that if the trend were allowed to continue, Easter (which is supposed to be a spring time feast) would move into summer, and then autumn, etc. The Pope set the zero point of the Gregorian calendar so that the vernal equinox would occur close to March 21. That was the date it had in the fourth century A.D. at the time of the Council of Nicaea, which had standardized the rule used to compute the date of Easter. This resetting of the zero point required the dropping of ten days from the calendar. By decree of the Pope, October 4 1582 was immediately followed by October 15 1582. The dates October 5 through October 14 1582 were simply dropped. They did not exist. (This caused rioting in several cities. People thought their lives were being shortened by 10 days.) The rule for Easter was modified at the same time to keep it in synchronism with the new leap year rule.

Of course, the Pope is Catholic. So his decree was universally ignored **except** in Roman Catholic countries and their colonies. However, religious beliefs could not alter the fact that the seasons were noticeably sliding. Gradually, Protestant countries adopted the Gregorian calendar. The dates varied from country to country. The later a country made the change, the more days had to be dropped. As European influence spread over the globe, most non-Christian countries adopted the Gregorian calendar, at least for civil use, as a matter of economic convenience. The last to make the change were, in general, Eastern Orthodox Christians. Today this calendar is the civil calendar nearly everywhere (but exceptions, especially in the Middle East, do still exist). Sample conversion dates: England—September 2 1752 was followed by September 14 1752; Japan in 1873; Turkey 1908; Greece converted in 1923. When England switched, it also began reckoning the start of a year as January 1.

The average length of a year in the Gregorian calendar is (400(365)+100-3)/400 = exactly 365.2425 days. This is in error (in the same direction as the error for the Julian Calendar) by about 0.0003 days per year, or one day in 3322 years.

A slight change to the Gregorian Calendar has been proposed which would improve the accuracy considerably. Under the modified rules, the leap year cycle would be 4000 years long. Years *whose position in the cycle* is divisible by 4 but not by 100 **AND** years whose position in the cycle is divisible by 400 but not by 4000 would be leap years. Thus 1900 A.D., 2100 A.D., and 101 B.C. would **not** be leap years (as with the present system). 401 B.C., 2000 A.D., and 3200 A.D. **would** be leap years (also the same as in the present system), but 1 B.C. and 4000 A.D. would **not** be leap years (different from the present system). To my knowledge no country has bothered to adopt this suggestion. Of course it will make no practical difference until the year 4000 A.D. This change, if adopted, would also force a modification in the rule for calculating the date of Easter. InterCal **does not** use the modified rules, but sticks to the traditional Gregorian

Calendar rules.

Western Historical

This “calendar” is not a true calendar at all. It is merely a concatenation of the Julian and Gregorian calendars, with the “stitch” point being October 4, 1582 (Julian Day number 2299159.5). Up to and including that day, the Julian Calendar rules are used. After that day, the Gregorian Calendar rules are used. Thus this calendar accurately represents historical dates (in the countries which adopted the Gregorian calendar as soon as it was decreed) from the stabilization of the Julian calendar forward. For Christian countries which waited to adopt the Gregorian Calendar until later, there is a period from October 15 1582 until their date of switching during which the Western Historical calendar is not historically accurate. Of course for non-Christian countries, the Julian Calendar was never used so the Western Historical calendar makes less sense. For fun, with the primary calendar system set to Western Historical (the default), display October 1582.

Jewish Calendar

The Jewish calendar is a mathematical luni-solar calendar. It evolved slowly over many, many centuries. During the Babylonian captivity the calendar took on several features of the Babylonian calendar, including the changing of most month names to their Babylonian equivalents. But long after the captivity the calendar continued to be improved. Many sources (most recently Sky and Telescope Magazine, September 1994 issue, “Hayyim Selig Slonimski”, pages 93–95) state that it reached its present form in 359 A.D., during the time of Patriarch Hillel II. But according to the Encyclopedia Judaica it did not reach its modern form until the tenth century A.D. Hillel II took a very important step, however. He made public the rules of the Jewish calendar, which until then had been a closely guarded secret known only to a few Jews and no Gentiles. The Encyclopedia Judaica speculates that Hillel II standardized the leap year rule (Rule #5 below). The more subtle features embodied in Rule 7f below apparently took a few hundred more years to become standardized.

The starting point of the Jewish calendar was intended by its developers to correspond to the creation of the world. Therefore, its zero point is quite far in the past and predates any written history. Not surprisingly, no era label for negative years ever developed. Negative years have not been needed. An era label for positive years exists, at least for use by scholars if not by the ordinary folks, but is rarely seen. It is A.M. (Anno Mundi, Latin for Year of the World). But InterCal needs era labels since it allows both positive and negative years. So I have invented (without intent to offend) the abbreviation B.W. (Before the World) and use A.M. even though it is not common practice.

The Jewish calendar month names and most terms used in its definition are Hebrew words. I know of no **standard** transliteration scheme for converting Hebrew into English. Therefore the English spellings of Jewish months, calendrical terms, and the names of religious holidays (such as Rosh Hashonah) are somewhat arbitrary. I have used the spellings found in the Jewish Calendar program (freeware written by Frank Yellin of Redwood City California). I wish to thank Mr. Yellin for making his source code available on the Internet. In spite of reading the article on the Jewish calendar in the Encyclopedia Judaica and in spite of pestering several Jewish friends, I was unable to develop a computer algorithm for this calendar until I examined Yellin's source. The "rules" below are my rendering of rules stated in the Encyclopedia Judaica as implemented by the Jewish Calendar program. My contributions in this area were to provide a faster implementation, remove Yellin's limits of dates corresponding to 1 A.D. through 2999 A.D., and extend the rules into negative years. I heartily recommend Yellin's program to anyone interested in the Jewish calendar. It has much more extensive labeling of Jewish religious festivals, holydays, etc. than InterCal.

The rules of the Jewish calendar, when stated (as below) in algorithmic form for use in a computer program, sound complicated and arbitrary. The rationale for the rules is explained in the Encyclopedia Judaica. To put them in the simplest terms—the 19-year leap year cycle with its implied cycle of months is followed exactly, and months start on or near days having a new moon. Most of the complexity is introduced because it is desired to prevent certain holy days (especially Yom Kippur) from falling on certain days of the week.

My rendering of the rules of the Jewish Calendar:

1) The months of a normal year are named (in order) Tishrei, Cheshvan, Kislev, Tevet, Shvat, Adar, Nissan, Iyar, Sivan, Tamuz, Ab, Elul. There are 12 months in normal years.

2) The months of a leap year are named (in order) Tishrei, Cheshvan, Kislev, Tevet, Shvat, Adar I, Adar II, Nissan, Iyar, Sivan, Tamuz, Ab, Elul. There are 13 months in leap years.

3) Month lengths are as follows: Tishrei, Shvat, Adar I, Nissan, Sivan, and Ab have 30 days; Tevet, Adar, Adar II, Iyar, Tamuz, and Elul have 29 days; Cheshvan and Kislev can each have either 29 or 30 days—their variable lengths are related to the special rules for determining the start of each year (1 Tishrei = Rosh Hashonah). The precise rule for their lengths can be found below (Rule #8).

4) The extra month added in leap years is Adar I. Adar and Adar II are actually the same month. Festivals which occur in Adar in normal years are observed in Adar II in leap years.

5) The leap year cycle is 19 years long. One cycle started with the year 1 A.M. and ended with the year 19 A.M. Another cycle started with the year 19 B.W. and ended with the year 1 B.W. Within any 19-year cycle, leap years occur in the third, sixth, eighth, eleventh, fourteenth, seventeenth, and nineteenth years. All other years in the cycle are normal years.

6) The Jewish calendar day begins at sunset, not midnight. InterCal follows the lead of the Encyclopedia Judaica and other sources in assigning weekdays to calendar dates as follows: If a calendar day begins

at sunset on a Friday and ends at sunset on a Saturday, that date is displayed as Saturday, and so forth for other days. But note that the times of day used in Rule #7 below are measured from 6:00 p.m.

7) The Julian Day of 1 Tishrei (Rosh Hashonah) of any year Y is determined as follows:

a) Given any non-zero integer Y, count the months elapsed from the start of the year 1 A.M. to the start of the year Y (using rules 1, 2, and 5 above). Since negative years are allowed, this number of months could be negative.

b) Multiply that number of months by the number of "parts" in a synodic month. "Parts" of a day are equal to exactly 3 1/3 seconds. There are 25920 parts per day. By **definition** (this is what ensures that the Jewish calendar is mathematical, not physical) a synodic month in the Jewish calendar contains exactly 765433 parts.

c) Add 5604 to the result of step b. (This is the time of day, in parts, at which by definition new moon occurred on or near 1 Tishrei of the year 1 A.M.)

d) Determine the number of **days** elapsed from the new moon associated with 1 Tishrei 1 A.M. to the new moon associated with 1 Tishrei in the year Y. That is, using integer division, divide the result from step c by 25920. Keep the remainder handy for use in later rules. That remainder is the time of day, in parts, of the new moon nearest the start of the year in question. (Integer division and modular arithmetic with negative numbers require care. Truncation must be towards minus infinity, and the remainder must always be non-negative. The lack of a year 0 must also be handled.)

e) The Julian Day at midnight during 1 Tishrei 1 A.M. (a Monday) is 347997.5. Add 347997.5 to the **integer** part of the result of step d. Determine the day of the week of this new moon.

f) Tentatively, the Julian Day at midnight during 1 Tishrei (Rosh Hashonah) of the year in question will be the just-calculated answer to step e. It represents the Julian Day at midnight of the day containing the new moon nearest the start of the year Y.

If necessary, adjust the date of Rosh Hashonah by one or two days using the following rules:

i) Determine (using rule 5) whether or not year Y is a leap year. Also determine whether or not the year **before** the year Y was a leap year.

ii) If the time of day of new moon is noon or later (the remainder in step d equals or exceeds 19440 parts) **OR**

iii) If the year Y is **not** a leap year and if the new moon is on a Tuesday and its time of day (remainder in step d) is greater than or equal to 9924 parts (9 hours plus 204 parts, or shortly after 3:00 a.m.) **OR**

iv) If the **preceding** year **is** a leap year and the new moon is on a Monday and its time of day is greater than or equal to 16789 parts (15 hours plus 589 parts, or a few minutes after 9:00 a.m.) **THEN**

Make the tentative day of Rosh Hashonah be the day **after** the day of new moon—that is, the new tentative Julian Day of Rosh Hashonah equals the answer to step e plus 1.

v) Set the actual Rosh Hashonah of the year in question to the tentative day (after possible adjustment using rules f.ii through f.iv) **unless** it would be a Sunday, Wednesday, or Friday. In those cases, make the actual Rosh Hashonah one day **after** the tentative day.

To summarize, the Julian Day at midnight during Rosh Hashonah is the answer to step 7e plus 0 or 1 or 2, depending on whether or not any of the conditions in steps 7.f.ii through 7.f.iv are true and whether or not the condition in step 7.f.v is true.

8) Determine the lengths of Cheshvan and Kislev as follows:

a) Subtract the Julian Day of Rosh Hashonah of the year Y from the Julian Day of Rosh Hashonah of the year **following** the year Y. Simply reapply Rule #7 in order to do this.

b) In spite of the apparent complexity of the rules, there are only six possible answers to step 8a. (Actually, the purpose of rules 7.f.ii through 7.f.v is to force these lengths to be the only ones possible.) The possible year lengths are: 353, 354, 355, 383, 384, and 385. Years of length 353 and length 383 are called “defective”. Years of length 354 and 384 are called “regular”. Years of length 355 and 385 are called “complete”. In defective years, both Cheshvan and Kislev have 29 days. In regular years, Cheshvan has 29 days and Kislev has 30. In complete years, both Cheshvan and Kislev have 30 days.

The average length of a Jewish month is 765433 parts, or 29.5305941… days for an error of 0.00000594 days per month. In 235 months (the number of months in each 19-year cycle) this error has built up to 0.00140 days for an average error of 0.0000735 days per year. Thus the Jewish calendar develops an error of one day with respect to the moon in approximately 13600 years (four times as accurate with respect to the moon as the Gregorian calendar is with respect to the sun). This is incredibly accurate for a calendar which is over one thousand years old in its present form! (Its month is slightly too long, so new moons will gradually slip backwards through the month as the millennia go by.)

On average, there are (235)29.5305941 days in each 19-year cycle or 365.246822 days per year. This is too long by 0.004623 days per year, or an error of one day every 216 years. This is slightly more accurate than the Julian calendar, but considerably less accurate than the Gregorian. Since the years are too long, actual astronomical events seem to slip backwards. Conversely, the calendar dates of events become later and later compared to the seasons with which they are supposed to be associated. As an example, in the present century Rosh Hashonah typically happens in September, with occasional forays into early October. 11000 years from now, given the present rules, Rosh Hashonah will have slipped nearly two months. It will occur from about 24 October through about 20 November.

Islamic Calendar

The Islamic calendar is a physical lunar calendar. That implies that any computer algorithm only approximates the true calendar. Therefore, going far into the past or the future is even less meaningful than for the mathematical calendars. Nevertheless, InterCal forges ahead!

In the true Islamic calendar, months start at the first official sighting of the new moon. Since it takes one to two days for a new moon to be visible after astronomical new moon. Islamic months are offset by a day or two from astronomical new moon. Note that this calendar is purely lunar, and makes no attempt at all to track the sun. Therefore the months cycle quickly through the seasons—it only takes 33 years for Ramadan (as an example) to cycle from a spring month all around the year and back to spring again.

The formulas used by InterCal (described below) are approximations to the Islamic calendar. I believe this system is in common use throughout most of the Islamic world for planning purposes.

The rules of the mathematical approximation to the Islamic calendar as implemented by InterCal are:

1) The negative year era is labeled B.H. (Before the Hegira). The positive year era is labeled A.H. (Anno Hegira, Year of the Hegira).

2) Months are named (in order) Muharram, Safar, Rabi‘ al-Awwal, Rabi‘ al-Akhir, Jumada’ al-Ula, Jumada’ al-Akhirah, Rajab, Sha’baan, Ramadan, Shawwal, Dhul-Qi’dah, Dhul-Hijjah. Years start in Muharram.

3) Days begin at sunset. The convention used for displaying days is the same as that used for the Jewish calendar, whose days also start at sunset. The Julian Day of midnight **during** Muharram 1, 1 A.H. Islamic (a Friday) is 1948439.5.

4) Muharram, Rabi‘ al-Awwal, Jumada’ al-Ula, Rajab, Ramadan, and Dhul-Qi’dah have 30 days.

5) Safar, Rabi‘ al-Akhir, Jumada’ al-Akhirah, Sha’baan, and Shawwal have 29 days.

6) Dhul-Hijjah has 29 days in normal years and 30 days in leap years.

7) The leap year cycle is thirty years long. One cycle began in the year 1 A.H. and ended in 30 A.H. Year #1 in the preceding cycle was 30 B.H. and year #30 in that cycle was 1 B.H. Leap years occur in years whose position in their cycle is 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, or 29.

With a leap year cycle of thirty years containing 354(30) + 11 days and 360 months, the average month is 29.53055556 days, for an error of 0.00003264 days per month, or approximately one day every 2500 years. The months are slightly too short. This **approximation** to the Islamic calendar has about the same error with respect to the moon that the Gregorian calendar has with respect to the sun. Remember that the official Islamic calendar is based on observations of the real moon, and so by definition has no perceptible error. Because the approximation’s months are too short, the first of each month computed by InterCal will come earlier and earlier compared to the actual start of the month as the millennia go by.

Elliott Super Calendar

The Elliott Super Calendar was invented as a toy by the author of InterCal. It is a mathematical luni-solar calendar. The eras are imaginatively named B.Z. and A.Z. for Before Zero and After Zero.

Just for fun, I decided to find a common multiple of the tropical year and the synodic month which was considerably closer than the 19-year (235-month) Metonic cycle discovered by the Babylonians and used in several other calendars (including the Jewish). I wanted accuracy, but did not want ridiculously long cycles of tens or hundreds of thousands of years. Although such long cycles might produce even greater accuracy, they would be unnecessarily complicated and not worthwhile in view of the long-term instability of the sun and moon’s motions. I found a very nice match with a 1689-year cycle containing 20890 months. This is only about four times as long as the Gregorian cycle, yet produces considerably greater accuracy for the sun and throws in the moon as well!

The Elliott Super Calendar uses astronomical new moon in its calculations. That is, it uses as definition of new moon the time at which the earth, moon, and sun are most nearly aligned in a straight line. The Jewish calendar evolved from a system in which actual sightings of a new crescent moon began each month. And the Islamic calendar continues to use such sightings for its official definition. Since a new crescent takes one or two days to become visible after astronomical new moon, the starts of months in the Jewish and Islamic calendars tend to be offset from those in the Elliott calendar by one or two days.

Taking a lesson from the Caesars, I named the first month after myself. Then I bested them by naming all the other months after my wife, children, parents, sisters, and my wife's parents and siblings.

For the zero point, I tried to find a place not too far away from the zero point of the Gregorian calendar. I also had two other criteria. For some year near the zero point I wanted new moon to occur very close to midnight at the start of the first day of the year. I also wanted the average date of the vernal equinox to be the first day of the year. (This rule was often used in ancient calendars, including the Babylonian and the Roman systems which pre-dated the Julian.) These considerations led to the following rules.

1) The negative year era is labeled B.Z. (Before Zero). The positive year era is labeled A.Z. (After Zero).

2) Months are named (in order) Denis, Jill, Nicole, Abigail, Ernest, Gladys, Gerard, Veronica, Anne, Colette, Joan, Ericka, and Anthony. Years start in Denis.

3) Normal years have 12 months (Denis through Ericka).

4) Leap years have 13 months (the month Anthony is added after Ericka).

5) Denis, Nicole, Ernest, Gerard, Anne, and Joan always have 30 days.

6) Jill, Abigail, Gladys, Veronica, Colette, and Ericka always have 29 days.

7) In leap years, the added month (Anthony) can have either 30 or 31 days. See Rules #8 and #10 below.

8) The leap year cycle is 1689 years long. One cycle began with 1 A.Z. and ended with 1689 A.Z. The cycle preceding that one began with 1689 B.Z. and ended with 1 B.Z. During each cycle there are three types of years. Normal years have 12 months. Leap Year Type 1 has the month Anthony added with Anthony having 30 days. Leap Year Type 2 has the month Anthony added with Anthony having 31 days. During each cycle there are 1067 normal years, 294 Type 1 leap years, and 328 Type 2 leap years. These leap years are distributed approximately evenly throughout the cycle.

9) The Julian Day of midnight at the start of Denis 1, 1 A.Z. Elliott (a Thursday) is 1751822.5. (This date corresponds to 25 March 84 A.D. Julian, which is 23 March 84 A.D. Gregorian.)

10) The rule for which years within each cycle are of which type is most easily understood in terms of a table. Such a table, having 1689 entries,

would specify the type of each year. But such a table is too long for this document. So instead, I specify the rule for computing that table. That is exactly what InterCal does during initialization.

a) Define two auxiliary tables, Type2 (with 328 entries) and Type 1 (with 294). For each integer N from 1 to 328, set its entry in Type2 to the nearest integer to 1689N/328 (**round**, don't truncate).

b) For each integer M from 1 to 294, set its entry in Type1 to the nearest integer to 1689M/294 - 3. (**Round**, don't truncate.) (The purpose of the "-3" is to offset entries in Type 1 from entries in Type2.) Occasionally during this process, the computed value of the M'th entry in Type1 will equal the (M-1)'th entry. Check for this, and when it happens go to the other table (Type2). Set the M'th entry of Type1 to the average of the two values in Type2 which are closest to, **but both larger than**, the duplicated value. When averaging, **truncate** to the next lowest integer if the average is not an integer.

c) Generate the final table, T1689, as follows:

i) For every integer J from 1 to 1689, set T1689(J) to "normal" **unless** J can be found in Type1 or Type2.

ii) If J is in Type1, set T1689(J) to "Leap Year Type 1".

iii) If J is in Type2, set T1689(J) to "Leap Year Type 2". **If computed correctly, there are no duplicates between or within the auxiliary tables.**

As samples, the first few entries in Type2 are 5, 10, 15, 21, 26, 31, 36, 41, 46, 51…
The first few entries in Type1 are 2, 7, 12, 18, 23, 28, 38, 43, 48, 54…

The rules above imply that there are exactly 616894 days in each 1689-year cycle. Thus the average length of a year is 365.2421551 days, so years are too short by 0.0000439 days, which amounts to an error of one day in 22780 years. This is nearly seven times as accurate as the Gregorian calendar and far more accurate than the Julian and Jewish calendars. The rules also imply that, on average, there are exactly 616894/20890 = 29.5305889 days per month. Thus the months are too long by 0.0000006 days. That is equivalent to an error of 0.012534 days per 1689-year cycle or 0.0000074 days per year. Thus, with respect to the moon, the Elliott calendar builds up an error of one day every 135000 years (approximately). This is ten times as accurate as the Jewish calendar, which (for the moon) is a very accurate calendar.

French Revolutionary Calendar

The French Revolutionary Calendar was instituted after the Revolutionaries achieved full control of France. It was the official calendar of France for a few years. In keeping with the passion of the Revolutionaries for rationality and naturalism, the calendar has every month the same length—30 days. The months are named for natural phenomena (mostly weather-related) such as Vintage, Hot, Rain, and Wind.

The French Revolutionary is a mathematical solar calendar. In fact, in spite of the renaming of months and offsetting of months and years, this calendar is based carefully on the Gregorian and is always kept synchronized with it. For that reason, its average year length and its accuracy with respect to the sun are exactly the same as for the Gregorian.

One oddity about the French Revolutionary calendar (especially considering who invented it) was that its years are specified in Roman numerals, not Arabic. InterCal takes the liberty of ignoring this, and states years in standard Arabic numerals as for all the other calendars. No era names appear to have been defined by the Revolutionaries. The era abbreviations used by InterCal are based on non-standardized but reasonably common usage of historians specializing in the period.

My thanks to Professor David Stewart of Hillsdale College for giving me the rules for this calendar.

The rules of the French Revolutionary calendar as implemented by InterCal, are:

1) The negative-year era is labeled A.R., for Ancien Régime. The positive-year era is labeled R.C. for Revolutionary Calendar.

2) Months are named, in order, Vendémiaire (Vintage), Brumaire (Fog), Frimaire (Sleet), Nivôse (Snow), Pluviôse (Rain), Ventôse (Wind), Germinal (Sprouting), Floréal (Flowering), Prairial (Pasturing), Messidor (Harvest), Thermidor (Hot), and Fructidor (Fruit). Years start in Vendémiaire.

3) The Julian Day of midnight at the start of Vendémiaire 1, 1 R.C. (a Saturday) is 2375839.5, which corresponds to September 22, 1792 A.D. Gregorian.

4) Every month has 30 days. Thus in most years Vendémiaire begins on September 22 in the Gregorian calendar; Brumaire begins on October 22; Frimaire on November 21, etc. Fructidor normally began on August 18 and ended on September 16. The leap year rule (item 7 below) changes this fixed relationship slightly.

5) After the end of Fructidor there are five extra days (in normal years) called jours complémentaires. In leap years a sixth extra day was added. Technically, these extra days were not considered to belong to any month. They each had a special name:

   a) Fête dès Vertus (The Virtues)—17 September Gregorian;
   b) Fête du Génie (Genius)—18 September;
   c) Fête du Travail (Labor)—19 September;
   d) Fête de l'Opinion (Opinion)—20 September;
   e) Fête dès Récompenses (Rewards)—21 September;
   f) In French Revolutionary leap years, a sixth extra day was

added, falling on September 22 and causing Vendémiaire 1 of the following year to be on September 23 Gregorian.

6) For implementation convenience, InterCal treats the jours complémentaires as a very short thirteenth month, containing five days in normal years and six in leap years. InterCal does not use the special names of the days, but simply numbers them.

7) The leap year cycle is four hundred years long, as in the Gregorian calendar. One cycle began in 1 R.C. and will end in 400 R.C. The preceding cycle began in 400 A.R. and ended in 1 A.R. French Revolutionary leap years are synchronized with Gregorian leap years as follows. French Revolutionary years which end in September of a year **preceding** a Gregorian leap year are French Revolutionary leap years (having the sixth extra day added). Thus French Revolutionary years which **contain** February 29 in the Gregorian calendar start one day late, but are not leap years. That is, Vendémiaire 1 corresponds to September 23 instead of the usual September 22. The months of Brumaire, Frimaire, Nivôse, Pluviôse, and Ventôse also start one day later than usual. February 29 Gregorian then falls on Ventôse 10, after which the French Revolutionary and Gregorian calendars are back in their usual synchronism. The fifth extra day at the end of such a year corresponds to September 21 Gregorian as always.

The above verbose and complicated rule reduces to the following strange-looking but concise mathematical rule: years **whose position in their cycle** equals 3 modulo 4 and does **not** equal 7 modulo 100, **and** year 207 in each cycle, are leap years.